ROBBINSVILLE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

Mathematics

Multivariable Calculus

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Course Philosophy

Every individual develops an understanding of the concepts of calculus and gains experience with its methods and applications. This understanding and experience is achieved through a multi-representational approach, which emphasizes the relationships between graphical, numerical, analytical, and verbal representations of concepts, problems, and solutions. The connections among the multiple representations of concepts and functions are strengthened by the regular use of technology by students and teacher. Technology also plays a key role in verifying written work and in aiding in the interpretation of experimentation results. Several unifying themes serve to bring cohesion to the course. These themes include derivatives, integrals, vectors, limits, approximation, and applications and modeling. They are developed using all of the major mathematical functions with which students are already familiar.

Course Description

Grade: 11- 12 Credits: 5 Course Length: 1 year Prerequisite: AP Calculus BC

Multivariable Calculus is a highly rigorous college level course. The concepts learned in single variable calculus will be extended to three dimensions. Topics studied include vector-valued functions, partial derivatives, multiple integrals, and vector calculus. Toward the end of the course, other additional topics will be explored including differential equations and the theorems of Gauss and Stokes. This course is weighted like an AP course.

Multivariable Calculus 7 E by James Stewart will serve as the main textbook.

Core and Supplemental Instructional Materials

Core Materials	Supplemental Materials
• Textbook: Multivariable Calculus 7 E by James Stewart	• Graphing websites (geogebra.com, desmos.com, etc.) • Graphing calculator • Online Resources • Secondary Textbook: Calculus: Multivariable 2nd Edition by Jon Rogawski

Integration of 21st Century Themes and Skills

Educational Technology
Standards: 8.1.12.A.2, 8.1.12.A.5
• **Technology Operations and Concepts:** Produce and edit a multi-page digital document for a commercial or professional audience and present it to peers and/or professionals in that related area for review. • **Technology Operations and Concepts:** Create a report from a relational database consisting of at least two tables, describe the process, and explain the report results. **Example:** Students can electronically submit a slide presentation on a research topic of their choosing and present it to their peers and teachers. They must include at least two tables and three graphs in their presentation and use them to help explain their results.

Career Ready Practices

Standards: (CRP1, CRP2, CRP4, CRP8, CRP11)

CRP1. Act as a responsible and contributing citizen and employee Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.

Example: Students will demonstrate the responsibilities associated with being a member of a community when engaging collaboratively during sharing in pairs/trios, and participating in whole group discussions. Examples may include jigsaw and fishbowl activities.

CRP2. Apply appropriate academic and technical skills.

Example: Students will demonstrate the skills learned in Multivariable Calculus when engaging collaboratively during sharing in pairs/trios and participating in whole group discussions. Examples may include jigsaw and fishbowl activities, as well as projects and formal assessments.

CRP4. Communicate clearly and effectively and with reason. Communication is a key factor in Multivariable Calculus. Students are aware that their words and techniques they use to convey their thoughts are crucial to audience understanding.

Example: Students will demonstrate clear and effective communication through written and oral assignments and assessments.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

Example: Students will demonstrate critical thinking as they determine the best methods to solve free response and multiple choice problems.

CRP11. Use technology to enhance productivity.

Example: Students will use technology to enhance productivity on a regular basis as they use graphing calculators and other graphing software to complete calculus problems.

Robbinsville Ready 21st Century Skill Integration

The following skills will be embedded throughout the curriculum and instruction of this course.

Collaborative Team Member: Robbinsville students will learn more by working together than in isolation. As educational theorist Lev Vygotsky advocated, learning is a social process. Many workplaces today encourage employees to work in teams to solicit diverse perspectives, brainstorm new ideas and/or products, and solve problems. Further, collaboration fosters interpersonal relationships, self-management skills, cooperation, and a sense of collective responsibility. Collaborative team members are able to work with diverse groups of people who hold a variety of perspectives.

Effective Communicator: Robbinsville students must be able to clearly articulate their ideas orally, in writing, and across various media in order to successfully connect to the world around them. As the world becomes increasingly globalized, communication is more than just sharing one's ideas. Effective communicators are able to communicate their convictions, actively listen and analyze others' work to identify perspective and/or potential bias.

Emotionally Intelligent Learner: Robbinsville students who are emotionally intelligent learn to be empathetic, demonstrate integrity and ethical behavior, are kind, are self-aware, willing to change, and practice self-care. They are better able to cope with the demands of the 21st century digital society and workplace because they are reliable, responsible, form stable and healthy relationships, and seek to grow personally and professionally. Emotionally intelligent people are able to manage their emotions, work effectively on teams and are leaders who can grow and help to develop others.

Informed and Involved Citizen: Robbinsville students need to be digital citizens who are civically and globally aware. The concept of what it means to be "literate" has evolved along with 21st century technological and cultural shifts. Our progressive vision of literacy entails having our students explore real world problems in the classroom. Informed and involved citizens are able to safely and accurately communicate with people all around the world and are financially, environmentally and informationally literate.

Innovative Thinker: Robbinsville students must encompass innovative thinking skills in order to be successful lifelong learners in the 21st century world. As stated by Karl Fisch and Scott McLeod in the short film Shift Happens, "We are currently preparing students for jobs that don't yet exist . . . using technologies that haven't been invented . . . in order to solve problems we don't even know are problems yet." Innovative thinkers are able to think analytically, solve problems critically, creatively engage in curiosity and tinkering, and demonstrate originality.

Resilient and Self-Directed Learner: Robbinsville students need to take risks and ultimately make independent and informed decisions in an ever-changing world. Author of Life, the Truth, and Being Free, Steve Maraboli stated, "Life doesn't get easier or more forgiving, we get stronger and more resilient." Self-directed scholars of the 21st century are able to set goals, initiate resolutions by seeking creative approaches, and adjust their thinking in light of difficult situations. Resilient students are able to take risks without fear of failure and overcome setbacks by utilizing experiences

to confront new challenges. Resilient and self directed scholars will consistently embrace opportunities to initiate solutions and overcome obstacles.

Interdisciplinary Connections

Multivariable Calculus connects widely to the following Career & Technical Education (CTE) Content Area: 21st Century Life and Careers Standards:

9.3.12.BM- MGT.2 Access, evaluate and disseminate information for business decision making.
Students will access, evaluate, and disseminate information for decision making on a regular basis as they apply the skills learned in Multivariable Calculus.

9.3.12.ED.2 Demonstrate effective oral, written, and multimedia communication in multiple formats and contexts.
Students will present oral and multimedia projects to the class in a wide variety of contexts throughout the course. They regularly communicate in writing as they practice application questions each unit.

9.3.12.AG-ENV.4 Develop proposed solutions to environmental issues, problems, and applications using scientific principles of meteorology, soil science, hydrology, microbiology, chemistry and ecology.
Many mathematical applications of Multivariable Calculus are geared towards environmental phenomena, such as rates of change of the flow of liquid, and other topics in physics (torque, work, force, flux, acceleration, etc.)

Robbinsville Public Schools
Scope, Sequence, Pacing and Assessment

Multivariable Calculus

Unit Title	Unit Understandings and Goals	Recommended Duration/ Pacing	Assessments			
			Formative	**Summative**	**Common Benchmark Assessments** (mid-course and end of course **only**)	**Alternative Assessments** (projects, etc. **when appropriate**)
Unit 1: Vectors and the Geometry of Space	• The geometry of three dimensional space, representing lines and planes in three dimensional space. • Constructing the equations of three dimensional figures in space, points, lines and planes as well as the equations of quadric surfaces. • Definition of a vector and the properties of vector addition and multiplication. • The dot product and cross product of two vectors can be used to create orthogonal projections, determine the angle between two vectors or create the perpendicular to the intersection of the vectors. • The cross product is the vector perpendicular to two given vectors, whose length is the area of the parallelogram determined by the vectors.	11 blocks	Unit Quizzes Open-Ended Questions Class Discussion In-class Assignments: (Group-work, Mini-Projects, Practice Material) Observations Exit Slips Quizzes Chapter Test	Unit Test with Varying Types of Questions Projects Authentic Assessments Core Assessments	Cumulative Exam Final Exam	Application Projects Graphing explorations Anticipatory Set/Warm Up Homework Quizzes Open-Ended Leading Questions Highlighting Prior Knowledge

Unit 2: Vector Functions	• Use the concept and properties for determining the derivative and integral of single variable functions to determine the derivative of vector functions. • Use computers (graphing calculators) to draw space curves given their functions. • Determine the curvature and arc length of a curve. Use vector functions to describe the motion of an object in space	8 blocks	Unit Quizzes Open-Ended Questions Class Discussion In-class Assignments: (Group-work, Mini-Projects, Practice Material) Observations Exit Slips Quizzes Chapter Test	Unit Test with Varying Types of Questions Projects Authentic Assessments Core Assessments	Cumulative Exam Final Exam	Application Projects Graphing explorations Anticipatory Set/Warm Up Homework Quizzes Open-Ended Leading Questions Highlighting Prior Knowledge
Unit 3: Partial Derivatives	• Use level curves to visual functions of several variables. • Extend the ideas of implicit differentiation from single variable calculus to determine particular, partial, derivatives for functions of several variables. • Find the limit of a multivariable function. • Use previous rules for differentiation, Chain Rule etc.to determine partial derivatives for functions of several variables. • Use fundamental concepts of limits and continuity to determine the limit and continuity for functions of several variables. • Determine tangent lines, planes and linear approximations of functions of several variables. • Use partial derivatives to create directional derivatives and the gradient vector of a function. • Use partial derivatives to determine	14 blocks	Unit Quizzes Open-Ended Questions Class Discussion In-class Assignments: (Group-work, Mini-Projects, Practice Material) Observations Exit Slips Quizzes Chapter Test	Unit Test with Varying Types of Questions Projects Authentic Assessments Core Assessments	Cumulative Exam Final Exam	Application Projects Graphing explorations Anticipatory Set/Warm Up Homework Quizzes Open-Ended Leading Questions Highlighting Prior Knowledge

	classifying characteristics of three dimensional surfaces, maximums, minimums, saddle points etc. ● Utilize Lagrange Multipliers in addition to partial derivatives to determine classifying characteristics of three dimensional surfaces					
Unit 4: Multiple Integrals	● Extend the ideas of integration from single variable calculus to determine multiple integrals for functions of several variables. ● Use double integrals to determine the volume of a three dimensional shape. ● Double integrals can be used to determine density, mass, moments and centers of mass, and probabilistic equations such as the joint density function. ● Use iterated integrals to determine the surface area of a quadric surface. ● The concept of multiple integrals can be extended from Cartesian Coordinates to Polar, Cylindrical and Spherical Coordinates. ● The substitution method from single variable calculus is extrapolated to the change of variables method in multiple integral to simplify integration.	12 blocks	Unit Quizzes Open-Ended Questions Class Discussion In-class Assignments: (Group-work, Mini-Projects, Practice Material) Observations Exit Slips Quizzes Chapter Test	Unit Test with Varying Types of Questions Projects Authentic Assessments Core Assessments	Cumulative Exam Final Exam	Application Projects Graphing explorations Anticipatory Set/Warm Up Homework Quizzes Open-Ended Leading Questions Highlighting Prior Knowledge
Unit 5: Vector Calculus	● Integrating a function over a curve C in place of a set interval [a, b] produces the line integral of the function. ● The fundamental theorem for line integrals relates a function's gradient and the integral of the function along a smooth curve. This creates the relationship that the line integral of the gradient of a function is the net change in the function. ● Green's Theorem provides the relationship between a line integral around a simple closed curve and a double integral over the	18 blocks	Unit Quizzes Open-Ended Questions Class Discussion In-class Assignments: (Group-work, Mini-Projects,	Unit Test with Varying Types of Questions Projects Authentic Assessments Core Assessments	Cumulative Exam Final Exam	Application Projects Graphing explorations Anticipatory Set/Warm Up Homework Quizzes

	plane bounded by the curve. ● The curl of a function is the vector field defined by the cross product of the function and its gradient. ● The divergence of a function is the scalar field defined by the dot product of the function and its gradient.		Practice Material) Observations Exit Slips Quizzes Chapter Test			Open-Ended Leading Questions Highlighting Prior Knowledge
Unit 6: Second-Order Differential Equations	● A second order linear differential equation is of the form P(x)((d^2 y)/(dx^2))+Q(x)(dy/dx)+R(x)y=G(x) where P,Q,R,and G are continuous functions. ● Second order differential equations are either homogeneous or nonhomogeneous. ● The method of undetermined coefficients is used to solve nonhomogeneous linear equations. ● Second Order Differential Equations are used to model various real-world situations, i.e. vibrations of strings and electric circuits. ● The solution to a second order differential equation can be represented as an infinite series	7 blocks	Unit Quizzes Open-Ended Questions Class Discussion In-class Assignments: (Group-work, Mini-Projects, Practice Material) Observations Exit Slips Quizzes Chapter Test	Unit Test with Varying Types of Questions Projects Authentic Assessments Core Assessments	Cumulative Exam Final Exam	Application Projects Graphing explorations Anticipatory Set/Warm Up Homework Quizzes Open-Ended Leading Questions Highlighting Prior Knowledge
Unit 7: Applications of Multivariable Calculus	● Multivariable Calculus can be applied to many real world situations, including: ○ Torque, Work, and Force ○ Velocity and Acceleration ○ Mass and Density ○ Electric Charge	8 blocks	Unit Quizzes Open-Ended Questions Class Discussion In-class Assignments:	Unit Test with Varying Types of Questions Project- design a lesson and teach the class	Cumulative Exam Final Exam	Application Projects Graphing explorations Anticipatory Set/Warm Up Homework

			(Group-work, Mini-Projects, Practice Material) Observations Exit Slips Quizzes			Quizzes Open-Ended Leading Questions Highlighting Prior Knowledge

Robbinsville Public Schools

Unit #1: Vectors and the Geometry of Space

Enduring Understandings:	**Essential Questions:**
• The geometry of three dimensional space, representing lines and planes in three dimensional space. • Constructing the equations of three dimensional figures in space, points, lines and planes as well as the equations of quadric surfaces. • Definition of a vector and the properties of vector addition and multiplication. • The dot product and cross product of two vectors can be used to create orthogonal projections, determine the angle between two vectors or create the perpendicular to the intersection of the vectors. • The cross product is the vector perpendicular to two given vectors, whose length is the area of the parallelogram determined by the vectors.	• How can the equations of functions in three dimensions be constructed visually? • How can we create equations for three dimensional space? • What are the defining characteristics for quadric surfaces and how are they determined? • What is the right hand rule for vectors and how does it apply in determining a solution?

Interdisciplinary Connection

Tech 8.2.12.A.3 The relationships among technologies and the connections between technology and other fields of study
Example: Students will use technology to graph quadric surfaces

Tech HS-ETS1-4. Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.
Example: Students will graph quadric surfaces on the computer

Guiding / Topical Questions with Specific Standards		**Content, Themes, Concepts, and Skills**	**Teaching Strategies**	**Instructional Resources and Materials**	**Assessment Strategies**
N-VM.A.1	How is a vector's magnitude and direction determined?	Determine the distance between two points in three dimensions.	Anticipatory sets to measure background knowledge and engage students Use guided and independent practice activities Use the Mimeo, whiteboard, and worksheets to reinforce the concepts Use cooperative learning activities	Worksheets and sample problems to analyze step-by-step solutions of the problems Textbook & associated materials Teacher created worksheets	Written tests and quizzes Worksheets Notebook assessments Response to discussion questions Anticipatory Sets/Do Now Problems
F-BF.A.1	How can three dimensional space be modeled by linear and polynomial functions?	Determine the equation of a line and plane in three dimensions.			

A-SSE.A.1	How can a dot product be used to create a scalar?	Determine the equation for a three dimensional object or quadric surface using defining characteristics.	Use discovery based learning activities that require students to make conjectures and investigate patterns Use whiteboards to show immediate feedback on questions	Mimeo lessons GeoGebra TI Smart View with TI 84 Graphing	Diagnostic Assessments to determine readiness Closure question/ Exit Slips
HSA-SSE.A.1	How can a cross product be used to determine the angle between two vectors?	Use the formula for dot product and cross product to determine missing values in a problem.			
A-CED.A.2 F-IF.C.7	How can you determine the second degree equation of a quadric surface?	Quadric surfaces are created from the equations of two dimensional conic sections.			

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Unit #2: Vector Functions

Enduring Understandings:	**Essential Questions**
• The concept and properties for determining the derivative and integral of single variable functions can be used to determine the derivative of vector functions. • Technology is helpful to draw space curves given their functions. • Vector functions can be used to describe the motion of an object in space, such as arc length and curvature.	• How are vector functions used to model motion through space? • How are previously developed concepts of calculus applied to vector functions? • How can vector functions be used to solve problems with real world applications?

Interdisciplinary Connection

Sci HS-PS2-1. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.
Example: Students will use vectors to model motion of an object

ELA NJSLSA.SL2. Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
Example: Students will use previously learned skills from Calculus to solve problems presented in new ways.

Guiding / Topical Questions with Specific Standards		**Content, Themes, Concepts, and Skills**	**Teaching Strategies**	**Instructional Resources and Materials**	**Assessment Strategies**
N-VM.A N-VM.B	How can vectors be used to model an object through space?	Perform algebraic operations, such as scalar multiplication, to manipulate vector functions.	Anticipatory sets to measure background knowledge and engage students Use guided and independent practice activities Use the Mimeo, whiteboard, and worksheets to reinforce the concepts Use cooperative learning activities Use discovery based learning activities that require students to make conjectures and investigate patterns Use whiteboards to show immediate feedback on questions	Worksheets and sample problems to analyze step-by-step solutions of the problems Textbook & associated materials Teacher created worksheets Mimeo lessons GeoGebra TI Smart View with TI 84 Graphing	Written tests and quizzes Worksheets Notebook assessments Response to discussion questions Anticipatory Sets/Do Now Problems Diagnostic Assessments to determine readiness Closure question/ Exit Slips
N-VM.B	How do the properties of single variable calculus allow for the differentiation and integration of vector functions?	Use rules for differentiation and integration from single variable calculus on vector value functions.			
N-VM.B G-C.A.4	What is the connection between vector functions and space curves?	Find the tangent and normal vectors to smooth curves at a given point.			

F-BF.A.1 G-C.B.5	How does the formula for the arc length and area of a circle extend to single variable and then vector functions?	Extrapolate the formula for arc length from single variable calculus to vector functions.			
F-LE.A	What values define the curvature of a smooth curve?	Use the concepts of tangent and normal vectors to determine the curvature of a smooth curve.			

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Unit #3: Partial Derivatives

Enduring Understandings:.	**Essential Questions**
• Level curves can be used to visual functions of several variables. • We can extend the ideas of implicit differentiation from single variable calculus to determine particular partial derivatives, limits, and continuity for functions of several variables. • Determine tangent lines, planes and linear approximations of functions of several variables. • Partial derivatives can be used to create directional derivatives, gradient vectors, extrema, and saddle points. • Lagrange Multipliers can be used in addition to partial derivatives to classify characteristics of three dimensional surfaces	• What is a function of several variables? • How do the concepts and rules of differentiation for functions of one variable extrapolate to functions of several variables? • How do you determine maximum, minimum and saddle points for functions of several variables? • What is the gradient of a function? • How does the method of Lagrange Multipliers work? • How can the limit of a multivariable function be shown to exist or not exist?

Interdisciplinary Connection

ELA RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
Example: Students will use multi-step methods to find extrema of three dimensional surfaces.

ELA RST.11-12.9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
Example: Students will use calculations, logic, and/or reasoning to determine extrema, and must determine the appropriate method to use based on their findings.

Tech HS-ETS1-4. Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.
Example: Students use technology to graph real world applications of a problem set and solutions

Guiding / Topical Questions with Specific Standards		**Content, Themes, Concepts, and Skills**	**Teaching Strategies**	**Instructional Resources and Materials**	**Assessment Strategies**
F-IF.C.7	What is a multivariable function and how is it visually represented?	Sketch several levels of the curves of a function to create a visualization.	Anticipatory sets to measure background knowledge and engage students Use guided and independent practice	Worksheets and sample problems to analyze step-by-step solutions of the problems	Written tests and quizzes Worksheets

A-CED.A.2 F-IF.C.7	If a function is continuous on a three dimensional coordinate system how is the graph of the function represented?	Use a contour map to develop a rough sketch of a function of several variables	activities Use the Mimeo, whiteboard, and worksheets to reinforce the concepts Use cooperative learning activities Use discovery based learning activities that require students to make conjectures and investigate patterns Use whiteboards to show immediate feedback on questions	Textbook & associated materials Teacher created worksheets Mimeo lessons GeoGebra TI Smart View with TI 84 Graphing	Notebook assessments Response to discussion questions Anticipatory Sets/Do Now Problems Diagnostic Assessments to determine readiness Closure question/ Exit Slips
F-BF.A.1 F-LE.A G-C.A.4	How is a tangent plane constructed for a quadric surface?	Use partial derivatives to create directional derivatives and the gradient vector of a function. Use previous rules for differentiation, Chain Rule etc. to determine partial derivatives for functions of several variables.			
A-SSE.A.1 F-LE.A	How can a linear approximation be used for a three dimensional function?	Use techniques of differentiation to determine the tangent planes and normal line of a given surface to a specified point or point parallel.			
A-CED.A.3 A-CED.A.4	What are the maximum, minimum and saddle points for functions of several variables?	Use partial derivatives to determine classifying characteristics of three dimensional surfaces, maximums, minimums, saddle points etc. Utilize Lagrange Multipliers in addition to partial derivatives to determine characteristics of three dimensional surfaces			

Unit #4: Multiple Integrals

Enduring Understandings:	**Essential Questions:**
• We can extend the ideas of integration from single variable calculus to determine multiple integrals for functions of several variables. • Double integrals can be used to determine the volume of a three dimensional shape and the surface area of a quadric surface. • Double integrals can be used to determine density, mass, moments and centers of mass, and probabilistic equations such as the joint density function. • The concept of multiple integrals can be extended from Cartesian Coordinates to Polar, Cylindrical and Spherical Coordinates. • The substitution method from single variable calculus is extrapolated to the change of variables method in multiple integral to simplify integration.	• What is the definition of a multiple integral? • How can an integral from Cartesian coordinates be converted to Polar, Cylindrical or Spherical coordinates? • What are the differences between a type I and type II regions? • What are the differences between a type I, II and III solid regions? • How are multiple and iterated integrals applied to real world problems?

Interdisciplinary Connection

ELA NJSLSA.R7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
Example: Students will use multiple integrals to solve problems presented by visual shapes, in words, or algebraically

ELA NJSLSA.W4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
Example: Students must be able to clearly write their reasoning for using the appropriate coordinate system.

Duration of Unit: 4 weeks

Guiding / Topical Questions with Specific Standards		**Content, Themes, Concepts, and Skills**	**Teaching Strategies**	**Instructional Resources and Materials**	**Assessment Strategies**
A-CED.A.4 MP.4 A-SSE.B.3	What is the benefit of changing from rectangular to polar coordinates for a double integral? What is the benefit of changing from rectangular to cylindrical, rectangular to spherical and spherical to cylindrical coordinates when evaluating an	When advantageous use the ability to convert between rectangular, polar, cylindrical and spherical coordinates to effectively and efficiently evaluate an iterated integral.	Anticipatory sets to measure background knowledge and engage students Use guided and independent practice activities Use the Mimeo, whiteboard, and worksheets to reinforce the concepts Use cooperative learning activities Use discovery based learning activities that require students to make	Worksheets and sample problems to analyze step-by-step solutions of the problems Textbook & associated materials Teacher created worksheets Mimeo lessons	Written tests and quizzes Worksheets Notebook assessments Response to discussion questions Anticipatory Sets/Do Now Problems Diagnostic

	iterated integral?		conjectures and investigate patterns Use whiteboards to show immediate feedback on questions	GeoGebra TI Smart View with TI 84 Graphing	Assessments to determine readiness Closure question/ Exit Slips
F-BF.A.1 G-GMD.A G-GMD.B G-MG.A	How are multiple integrals used to define geometric spaces? What happens to the value of a multiple integral if the function takes on both positive and negative values? How is a double integral defined if the function is not rectangular?	Describe the region given by a definite iterated integral and describe the volume of the solid generated by evaluating the integral. Use change of variables and appropriate transformations to evaluate multiple integrals.			

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Unit #5: Vector Calculus

Enduring Understandings:	**Essential Questions: :**
• Integrating a function over a curve C in place of a set interval [a, b] produces the line integral of the function. • The fundamental theorem for line integrals relates a function's gradient and the integral of the function along a smooth curve. This creates the relationship that the line integral of the gradient of a function is the net change in the function. • Green's Theorem provides the relationship between a line integral around a simple closed curve and a double integral over the plane bounded by the curve. • The curl of a function is the vector field defined by the cross product of the function and its gradient. • The divergence of a function is the scalar field defined by the dot product of the function and its gradient.	• What is a vector field? • What is the fundamental theorem for line-integrals? • What is Green's Theorem? • What is the relationship between curl and divergence? • What is the relationship between the Fundamental Theorem of Calculus, Fundamental Theorem for Line Integrals, Green's Theorem, and Stoke's Theorem and the Divergence Theorem?

Interdisciplinary Connection

Sci HS-PS2-3. Apply scientific and engineering ideas to design, evaluate, and refine a device that minimizes the force on a macroscopic object during a collision.
Sci HS-PS2-5. Plan and conduct an investigation to provide evidence that an electric current can produce a magnetic field and that a changing magnetic field can produce an electric current.
Sci HS-LS2-4. Use mathematical representations to support claims for the cycling of matter and flow of energy among organisms in an ecosystem.
Example: Students will use line integrals to analyze flux, force, and flow of fluid through objects

ELA NJSLSA.R7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
Example: Students will interpret problems that are presented in many different ways, depending on the theorem used.

Guiding / Topical Questions with Specific Standards		**Content, Themes, Concepts, and Skills**	**Teaching Strategies**	**Instructional Resources and Materials**	**Assessment Strategies**
A-CED.A.4 MP.4 A-SSE.B.3 G-C.B.5	How does a line integral relate to the arc length of curve? How can a line integral be used to approximate mass, density and center of mass?	Evaluate the line integral of a smooth, simple, closed plane curve of a piecewise function under basic operations. Evaluate the line integral of function as applied to various geometric situations and oriented surfaces.	Anticipatory sets to measure background knowledge and engage students Use guided and independent practice activities Use the Mimeo, whiteboard, and worksheets to reinforce the	Worksheets and sample problems to analyze step-by-step solutions of the problems Textbook & associated materials	Written tests and quizzes Worksheets Notebook assessments Response to discussion

	What is the relationship between the line integral of a composite function and the line integrals of its components?		concepts Use cooperative learning activities Use discovery based learning activities that require students to make conjectures and investigate patterns Use whiteboards to show immediate feedback on questions	Teacher created worksheets Mimeo lessons GeoGebra TI Smart View with TI 84 Graphing	questions Anticipatory Sets/Do Now Problems Diagnostic Assessments to determine readiness Closure question/ Exit Slips
N-VM.A N-VM.B	What is a vector field? A conservative field? A potential function? How is a vector field determined to be conservative in three dimensions?	Given a vector field, curve and a point determine if the line integral of the function is positive, negative or zero. Show that F is a conservative vector field and determine the function. Use a conservative vector field to evaluate a line integral.			
F-BF.A.1	How is a surface integral defined over an oriented surface with a unit normal vector?	Use Green's Theorem, Stokes' Theorem and the Divergence Theorem as applied to various geometric shapes and oriented surfaces.			

Robbinsville Public Schools

Unit #6: Second-Order Differential Equations

Enduring Understandings:	**Essential Questions: :**
• A second order linear differential equation is of the form P(x)((d^2 y)/(dx^2))+Q(x)(dy/dx)+R(x)y=G(x) where P, Q, R, and G are continuous functions. • Second order differential equations are either homogeneous or nonhomogeneous. • The method of undetermined coefficients is used to solve nonhomogeneous linear equations. • Second Order Differential Equations are used to model various real-world situations, i.e. vibrations of strings and electric circuits. • The solution to a second order differential equation can be represented as an infinite series	• What is the general form of the auxiliary equation for a second order differential equation? • How do the roots of the auxiliary equations affect the form of the solution to a second order-differential equation? • How is an initial value problem solved? • How are power series used to solve a differential equation?

Interdisciplinary Connection

Sci HS-PS3-5. Develop and use a model of two objects interacting through electric or magnetic fields to illustrate the forces between objects and the changes in energy of the objects due to the interaction.
Example: Students will use second order differential equations to calculate electric charge

ELA **RST.11-12.3.** Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
Example: Students will use multiple steps when solving non homogeneous second order differential equations

Guiding / Topical Questions with Specific Standards		**Content, Themes, Concepts, and Skills**	**Teaching Strategies**	**Instructional Resources and Materials**	**Assessment Strategies**
A-CED.A.4 A-SSE.B.3 F-BF.A.1 N-CN.B.4	What are the general forms of homogeneous second order linear differential equations with constant coefficients? How are the roots of an auxiliary equation used to determine the form of the	Solve the second order differential equation using an auxiliary equation, characteristic equation, method of variation of parameters for the general and specific solutions. Apply second order differential equations to real world value problems such as vibration and circuits to solve for initial and approximate values.	Anticipatory sets to measure background knowledge and engage students Use guided and independent practice activities Use the Mimeo, whiteboard, and worksheets to reinforce the concepts Use cooperative learning activities Use discovery based learning activities	Worksheets and sample problems to analyze step-by-step solutions of the problems Textbook & associated materials Teacher created worksheets Mimeo lessons	Written tests and quizzes Worksheets Notebook assessments Response to discussion questions Anticipatory Sets/Do Now Problems

	solution for a second order differential equation? What are the general forms of nonhomogeneous second order linear differential equations with constant coefficients?		that require students to make conjectures and investigate patterns Use whiteboards to show immediate feedback on questions	GeoGebra TI Smart View with TI 84 Graphing	Diagnostic Assessments to determine readiness Closure question/ Exit Slips
A-CED.A.4 F-BF.A.1	How can power series be used to evaluate initial value problems of second order differential equations?	Power series can be used to approximate variable coefficients in the general form solution of both homogeneous and nonhomogeneous second order differential equations			

Robbinsville Public Schools

Unit #7: Applications of Multivariable Calculus

Enduring Understandings:	**Essential Questions: :**
● Multivariable Calculus can be applied to many real world situations, including: ○ Torque, Work, and Force ○ Velocity and Acceleration ○ Mass and Density ○ Electric Charge	● How can we apply multivariable calculus to real world situations? ● How can I design an effective lesson to teach my classmates?

Interdisciplinary Connection

Tech 8.1.12.E.1 Evaluate and select information sources and digital tools based on the appropriateness for specific tasks.
Example: Students will use technology to present a lesson to the class

Tech 9.3.12.ED.2 Demonstrate effective oral, written, and multimedia communication in multiple formats and contexts.
Example: Students will present oral and multimedia projects to the class. They will communicate in writing through the notes and worksheets they create.

ELA RST.11-12.7 Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.
Example: Students will use textbooks or other sources to create and present a lesson to the class

ELA NJSLSA.W7. Conduct short as well as more sustained research projects, utilizing an inquiry-based research process, based on focused questions, demonstrating an understanding of the subject under investigation.
Example: Students will write notes, worksheets, and quizzes to demonstrate their understanding of the applications of Calculus.

Guiding / Topical Questions with Specific Standards		**Content, Themes, Concepts, and Skills**	**Teaching Strategies**	**Instructional Resources and Materials**	**Assessment Strategies**
MP.4 N-Q.A	How can vectors be applied to real world situations involving velocity and acceleration? How can we use calculus to find torque, work, and force?	Use previously learned skills in this course to solve real-world problems.	Anticipatory sets to measure background knowledge and engage students Use guided and independent practice activities Use the Mimeo, whiteboard, and worksheets to reinforce the concepts Use cooperative learning activities	Worksheets and sample problems to analyze step-by-step solutions of the problems Textbook & associated materials Teacher created worksheets	Written tests and quizzes Worksheets Notebook assessments Response to discussion questions Anticipatory Sets/Do

MP.2	How are second order differential equations used in electrical science?	Find electric charge using second order differential equations	Use discovery based learning activities that require students to make conjectures and investigate patterns Use whiteboards to show immediate feedback on questions	Mimeo lessons GeoGebra TI Smart View with TI 84 Graphing	Now Problems Diagnostic Assessments to determine readiness Closure question/ Exit Slips
MP.4 G-MG.A	How are iterated integrals used to calculate the density, mass, moment of inertia etc. of a function?	Use iterated integrals to determine qualities, mass, density, etc. of lamina in three dimensional space.			

General Differentiated Instruction Strategies	
• Leveled texts • Chunking texts • Choice board • Socratic Seminar • Tiered Instruction • Small group instruction • Guided Reading • Sentence starters/frames • Writing scaffolds • Tangible items/pictures • Adjust length of assignment	• Repeat, reword directions • Brain breaks and movement breaks • Brief and concrete directions • Checklists for tasks • Graphic organizers • Assistive technology (spell check, voice to type) • Study guides • Tiered learning stations • Tiered questioning • Data-driven student partnerships • Extra time

Possible Additional Strategies for Special Education Students, 504 Students, At-Risk Students, and English Language Learners (ELLs)			
Time/General	**Processing**	**Comprehension**	**Recall**

• Extra time for assigned tasks • Adjust length of assignment • Timeline with due dates for reports and projects • Communication system between home and school • Provide lecture notes/outline	• Extra Response time • Have students verbalize steps • Repeat, clarify or reword directions • Mini-breaks between tasks • Provide a warning for transitions • Reading partners	• Precise step-by-step directions • Short manageable tasks • Brief and concrete directions • Provide immediate feedback • Small group instruction • Emphasize multi-sensory learning	• Teacher-made checklist • Use visual graphic organizers • Reference resources to promote independence • Visual and verbal reminders • Graphic organizers
Assistive Technology	**Assessments and Grading**	**Behavior/Attention**	**Organization**
• Computer/whiteboard • Tape recorder • Spell-checker • Audio-taped books	• Extended time • Study guides • Shortened tests • Read directions aloud	• Consistent daily structured routine • Simple and clear classroom rules • Frequent feedback	• Individual daily planner • Display a written agenda • Note-taking assistance • Color code materials

Enrichment

The goal of Enrichment is to provide learners with the opportunity to participate in extension activities that are differentiated and enhance the curriculum. All enrichment decisions will be based upon individual student needs.

- Show a high degree of intellectual, creative and/or artistic ability and demonstrate this ability in multiple ways.
- Pose questions and exhibit sincere curiosity about principles and how things work.
- The ability to grasp concepts and make real world and cross-curricular connections.
- Generate theories and hypotheses and pursue methods of inquiry.
- Produce products that express insight, creativity, and excellence.
- Possess exceptional leadership skills.
- Evaluate vocabulary
- Elevate Text Complexity
- Inquiry based assignments and projects

- Independent student options
- Tiered/Multi-level activities
- Purposeful Learning Center
- Open-ended activities and projects
- Form and build on learning communities
- Providing pupils with experiences outside the 'regular' curriculum
- Altering the pace the student uses to cover regular curriculum in order to explore topics of interest in greater depth/breadth within their own grade level
- A higher quality of work than the norm for the given age group.
- The promotion of a higher level of thinking and making connections.
- The inclusion of additional subject areas and/or activities (cross-curricular).
- Using supplementary materials in addition to the normal range of resources.

English Language Learner (ELL) Resources

- Learning style quiz for students- http://www.educationplanner.org/students/self-assessments/learning-styles-quiz.shtml
- "Word clouds" from text that you provide-http://www.wordle.net/
- Bilingual website for students, parents and educators: http://www.colorincolorado.org/
- Learn a language for FREE-www.Duolingo.com
- Time on task for students-http://www.online-stopwatch.com/
- Differentiation activities for students based on their Lexile-www.Mobymax.com
- WIDA-http://www.wida.us/
- Everything ESL - http://www.everythingESL.net
- ELL Tool Box Suggestion Site Http://www.wallwisher.com/wall/ell toolbox
- Hope4Education - http://www.hope4education.com
- Learning the Language http://blogs.edweek.org/edweek/learning-the-language/
- FLENJ (Foreign Language Educators of NJ) 'E-Verse' wiki: http://www.flenj.org/Publications/?page=135
- OELA - http://www.ed.gov/offices/OBEMLA
- New Jersey Department of Education- Bilingual Education information http://www.state.nj.us/education/bilingual/

Special Education Resources

- Animoto -Animoto provides tools for making videos by using animation to pull together a series of images and combining with audio. Animoto videos or presentations are easy to publish and share. https://animoto.com
- Bookbuilder -Use this site to create, share, publish, and read digital books that engage and support diverse learners according to their individual needs, interests, and skills. http://bookbuilder.cast.org/
- CAST -CAST is a non-profit research and development organization dedicated to Universal Design for Learning (UDL). UDL research demonstrates that the challenge of diversity can and must be met by making curriculum flexible and responsive to learner differences. http://www.cast.org
- CoSketch -CoSketch is a multi-user online whiteboard designed to give you the ability to quickly visualize and share your ideas as images. http://www.cosketch.com/
- Crayon -The Crayon.net site offers an electronic template for students to create their own newspapers. The site allows you to bring multiple sources together, thus creating an individualized and customized newspaper. http://crayon.net/ Education Oasis -Education Oasis offers a collection of graphic organizers to help students organize and retain knowledge – cause and effect, character and story, compare and contrast, and more! http://www.educationoasis.com/printables/graphic-organizers/
- Edutopia -A comprehensive website and online community that increases knowledge, sharing, and adoption of what works in K-12 education. We emphasize core strategies: project-based learning, comprehensive assessment, integrated studies, social and emotional learning, educational leadership and teacher development, and technology integration. http://www.edutopia.org/
- Glogster -Glogster allows you to create "interactive posters" to communicate ideas. Students can embedded media links, sound, and video, and then share their posters with friends. http://edu.glogster.com/?ref=personal
- Interactives – Elements of a Story -This interactive breaks down the important elements of a story. Students go through the series of steps for constructing a story including: Setting, Characters, Sequence, Exposition, Conflict, Climax, and Resolution. http://www.learner.org/interactives/story/index.html
- National Writing Project (NWP) -Unique in breadth and scale, the NWP is a network of sites anchored at colleges and universities and serving teachers across disciplines and at all levels, early childhood through university. We provide professional development, develop resources, generate research, and act on knowledge to improve the teaching of writing and learning in schools and communities. http://www.nwp.org
- Pacecar -Vocab Ahead offers videos that give an active demonstration of vocabulary with audio repeating the pronunciation, definition, various uses, and synonyms. Students can also go through flash cards which give a written definition and visual representation of the word. http://pacecar.missingmethod.com/