

**Robbinsville High School**  
**Mathematics Department**  
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Robbinsville NJ 08691

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Dear Students,

Welcome to AP Calculus! Attached you will find a summer packet for math reinforcement for the upcoming school year. This packet should be completed and returned to school on the *first full day of school*. The packet will be **collected** and **graded** as a **20-point homework grade** based on **completion** and **effort**. Work is required for many of these problems, so unsupported answers will not receive credit.

The packet covers material from Honors PreCalc that is found in Chapters 1 and 2 and Section 3.1 in your textbook. You will probably need to use your textbook or other resources in order to complete this packet. In addition to completing the summer assignment, please review all material in Chapter 1, 2 and 3.1. **Your first test will be on the THIRD day of school** and will cover material from this packet *and* Chapters 1,2 and Section 3.1.

The packet itself is only a sampling of concepts and questions that are prerequisite for entering AP Calc. In addition the packet, supplemental problems for the topics can be found below. *Please note that these are **extra** problems and are not part of the required summer work and will not be graded.*

|  |   |
|--|---|
| Section 2.1 Rates of Change and Limits   | Problems 3-30 multiples of 3, 33, 36, 38, 47, 51-58, 61, 64 |
| Section 2.2 Limits at Infinity           | Problems 3-54 multiples of 3, 53-54, 59, 68, 70             |
| Section 2.3 Continuity                   | Problems 3-30 multiples of 3, 42, 45, 51, 52, 63            |
| Section 2.4 Rates of Change and Tangents | Problems 1-33 odd, 26, 30, 32, 47                           |
| Section 3.1 Derivative of a Function     | Problems 1-35 odd, 42                                       |

If you have any questions while completing the packet, please feel free to email me over the summer at [sawin@robbinsville.k12.nj.us](mailto:sawin@robbinsville.k12.nj.us) .

Have a great summer!  
Ms. Sawin

## PreCalc Review (Ch. 1)

### NON-CALCULATOR PORTION

Composite Functions: use the given information to determine the missing function

1)  $f(x) = (x+3)^2$        $g(x) = \frac{1}{x}$

a)  $f(g(x)) =$  \_\_\_\_\_

b)  $g(g(3)) =$  \_\_\_\_\_

2)  $f(g(x)) = \sqrt{x^3 - 6}$        $g(x) = x^2$

$f(x) =$  \_\_\_\_\_

3. Let  $y = 3 \cos(4x - \pi) - 5$ . Fill in the information below.

Period \_\_\_\_\_ Amplitude \_\_\_\_\_

Horizontal Shifts \_\_\_\_\_

Vertical Shifts \_\_\_\_\_

Determine whether the following functions have an inverse function.

If yes, find an equation for the inverse.

4.  $y = (x-1)^3 + 5$

5.  $y = x^2 + x - 2$

6. Evaluate the following trig values:

a)  $\cos \frac{\pi}{2} =$  \_\_\_\_\_

b)  $\tan \frac{3\pi}{4} =$  \_\_\_\_\_

c)  $\sin\left(-\frac{\pi}{3}\right) =$  \_\_\_\_\_

d)  $\sin \frac{3\pi}{2} =$  \_\_\_\_\_

e)  $\cot \pi =$  \_\_\_\_\_

f)  $\csc \frac{11\pi}{6} =$  \_\_\_\_\_

g)  $\tan \frac{5\pi}{3} =$  \_\_\_\_\_

h)  $\sec \frac{\pi}{4} =$  \_\_\_\_\_

i)  $\tan\left(-\frac{\pi}{2}\right) =$  \_\_\_\_\_

7. Evaluate the following. Give all answers in radians.

a)  $\arccos(1)$  \_\_\_\_\_

b)  $\arcsin(-1)$  \_\_\_\_\_

c)  $\arctan\left(\frac{\sqrt{3}}{3}\right)$  \_\_\_\_\_

d)  $\arctan(-1)$  \_\_\_\_\_

e)  $\arccos\left(-\frac{\sqrt{2}}{2}\right)$  \_\_\_\_\_

f)  $\arcsin\left(-\frac{1}{2}\right)$  \_\_\_\_\_

g)  $\arctan(0)$  \_\_\_\_\_

h)  $\arccos\left(\frac{1}{2}\right)$  \_\_\_\_\_

i)  $\arccos\left(-\frac{\sqrt{3}}{2}\right)$  \_\_\_\_\_

j)  $\arcsin\left(\sin \frac{2\pi}{3}\right)$  \_\_\_\_\_

k)  $\sin(\arcsin(-.25))$  \_\_\_\_\_

l)  $\arctan\left(\tan\left(-\frac{\pi}{6}\right)\right)$  \_\_\_\_\_

m)  $\arccos\left(\cos\left(\frac{3\pi}{2}\right)\right)$  \_\_\_\_\_

n)  $\cos(\arccos(-5))$  \_\_\_\_\_

o)  $\arctan\left(\tan\left(\frac{7\pi}{6}\right)\right)$  \_\_\_\_\_

## CALCULATOR PORTION

7. Money is deposited into an account that earns 7% interest, compounded annually. You will not earn the interest until the calendar year is over. How long will it take for the balance to double? Show the work that leads to your answer.

8. Use the functions below to answer the following questions. Show all work or give an explanation. **You may use your calculator to check yourself; however, "I used the calculator" is not proper justification for an answer.**

$$F(x) = 2x^2 + 1$$

$$g(x) = 3^x - 4$$

$$h(x) = -1 + \sqrt{1 - x^2}$$

a) Is  $h(x)$  even, odd, or neither? Justify your answer.

b) Find  $g(f(1))$

c) Find the zero(s) of  $g(x)$ .

d) Find the inverse of  $g(x)$ .

e) Determine the domain and range of  $f(x)$

f) Determine the domain and range of  $g(x)$

g) Determine the domain and range of  $h(x)$

## Section 2.1: Rates of Change and Limits

Find the limits below. Be sure to show all work and give exact answers.

1)  $\lim_{x \rightarrow 0} \frac{x^2}{x+5} =$

2)  $\lim_{x \rightarrow 5} 6 =$

3)  $\lim_{x \rightarrow 3} \frac{(x-4)^2}{x+3} =$

4)  $\lim_{x \rightarrow 0} 3x \cos x =$

5)  $\lim_{x \rightarrow 0} \frac{\sin 3x}{6x} =$

6)  $\lim_{x \rightarrow -2} \frac{x^2 + x - 2}{x^2 + 5x + 6} =$

7)  $\lim_{x \rightarrow 7} \frac{\sqrt{2x-5} - 3}{x-7} =$

8)  $\lim_{x \rightarrow 0} \frac{\frac{1}{(x+5)} - \frac{1}{5}}{x} =$

9)  $\lim_{x \rightarrow 0} (\ln(\cos(x))) =$

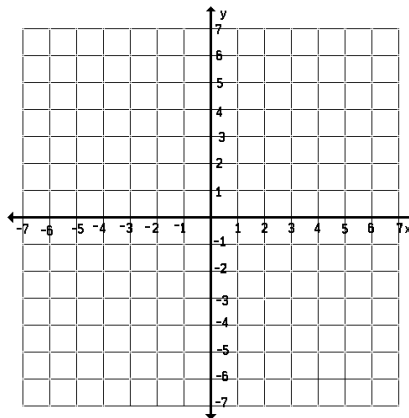
10)  $\lim_{x \rightarrow \pi/2} (e^x \sin(x)) =$

11)  $\lim_{x \rightarrow 1^-} \text{int}(x) =$

12)  $\lim_{x \rightarrow 5} \frac{x^2 - 7x + 10}{x-5} =$

13) Using the piecewise function, graph and then find the limits:

$$f(x) = \begin{cases} x-2 & x < 1 \\ 3 & x = 1 \\ 5-x & x > 1 \end{cases}$$



a)  $\lim_{x \rightarrow 1^+} f(x) =$

b)  $\lim_{x \rightarrow 1^-} f(x) =$

c)  $\lim_{x \rightarrow 1} f(x) =$

d)  $f(1) =$

14) Use the limits  $\lim_{x \rightarrow 3} f(x) = 5$  and  $\lim_{x \rightarrow 3} g(x) = -2$  to answer the following:

a)  $\lim_{x \rightarrow 3} f(x) + g(x) =$

b)  $\lim_{x \rightarrow 3} f(x) \cdot g(x) =$

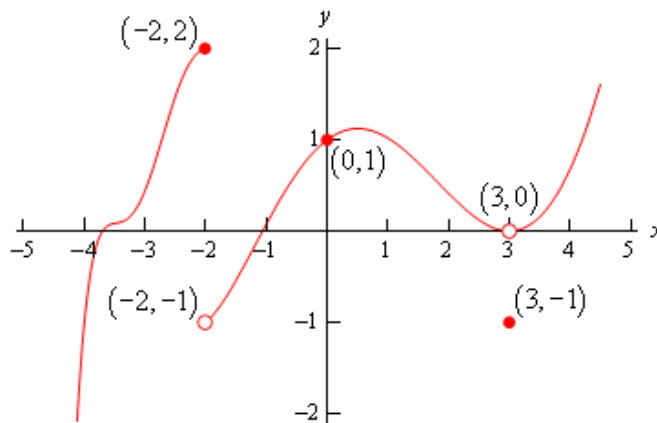
c)  $\lim_{x \rightarrow 3} 3f(x) - g(x) =$

d)  $\lim_{x \rightarrow 3} \frac{f(x) - 5}{g(x)} =$

15) A cookie dropped from a window falls  $y = 4.9t^2$  meters in  $t$  seconds. Find the average rate of change of the cookie over the first 4 seconds. Round your speed to 1 decimal place.

16) The same cookie from problem 15 still falls  $y = 4.9t^2$  meters in  $t$  seconds. Find the instantaneous rate of change at 3 seconds. Round your speed to 1 decimal place.

17) Use the following diagram to answer the questions:



a)  $f(3) =$

b)  $\lim_{x \rightarrow 3^-} f(x) =$

c)  $\lim_{x \rightarrow 3^+} f(x) =$

d)  $\lim_{x \rightarrow 3} f(x) =$

e)  $f(-2) =$

f)  $\lim_{x \rightarrow -2^-} f(x) =$

g)  $\lim_{x \rightarrow -2^+} f(x) =$

h)  $\lim_{x \rightarrow -2} f(x) =$

i)  $f(0) =$

j)  $\lim_{x \rightarrow 0^-} f(x) =$

k)  $\lim_{x \rightarrow 0^+} f(x) =$

l)  $\lim_{x \rightarrow 0} f(x) =$

m)  $\lim_{x \rightarrow \infty} f(x) =$

n)  $\lim_{x \rightarrow -\infty} f(x) =$

## Section 2.2: Limits Involving Infinity

Find the limits below.

$$1) \lim_{x \rightarrow \infty} \ln x =$$

$$2) \lim_{x \rightarrow -\infty} e^{-x} =$$

$$3) \lim_{x \rightarrow \infty} \frac{4x^4 - 5x^3}{7x^4 + 9x^3} =$$

$$4) \lim_{x \rightarrow \infty} \frac{3x^3 - x + 1}{x + 3} =$$

$$5) \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 - 2}}{x^2 + 6} =$$

$$6) \lim_{x \rightarrow \infty} \frac{1 - 7x^2}{x + 5} =$$

$$7) \lim_{x \rightarrow 0} \frac{\sin x}{5x}$$

$$8) \lim_{x \rightarrow 0} \frac{3(1 - \cos x)}{x}$$

$$9) \lim_{x \rightarrow 0} \frac{\cos x \tan x}{x}$$

$$10) \lim_{x \rightarrow 2} \frac{3x^2 - 7x + 2}{x^2 + 5x - 14}$$

$$11) \lim_{x \rightarrow 0} \frac{x^2 - 4}{x + 2}$$

$$12) \lim_{x \rightarrow \infty} \frac{2x + \sin x}{x}$$

$$13) \lim_{x \rightarrow \infty} \frac{x - 6}{x^2 + 2x - 48}$$

$$14) \lim_{x \rightarrow -\infty} \frac{x^3 + 6x}{\sqrt{x^2 + 5}}$$

$$15) \lim_{x \rightarrow -4^+} \frac{1}{x + 4}$$

$$16) \lim_{x \rightarrow -\infty} \sqrt[3]{\frac{8 + x^2}{8x(x + 1)}}$$

$$17) \lim_{x \rightarrow 4} \frac{\sqrt{x^2 + 9} + -5}{x - 4}$$

$$18) \lim_{x \rightarrow 0} \frac{\tan x}{x}$$

### Section 2.3: Continuity

1) Find all points of discontinuity of the functions below and state the type of discontinuity. If the function has no points of discontinuity, then specify over what intervals it is continuous.

a)  $f(x) = \frac{x+1}{x^2-4}$

b)  $f(x) = \frac{x^2-8x+15}{x^2-25}$

c)  $f(x) = 3x+9$

d)  $f(x) = \sqrt{2x-7}$

e)  $f(x) = \frac{8-2x}{x^2-16}$

f)  $f(x) = \frac{x}{|x|-3}$

2) At what x-coordinate on  $f(x) = \frac{x^2 - x - 6}{x^2 - 9}$  is there a removable discontinuity?

3) Find a value for  $a$  so that function is continuous.

$$\text{a) } f(x) = \begin{cases} 4 - x^2, & x < -1 \\ ax^2 - 1, & x \geq -1 \end{cases}$$

$$\text{b) } f(x) = \begin{cases} x^2 + x + a, & x < 1 \\ x^3, & x \geq 1 \end{cases}$$

4) Given the information below, sketch a possible graph of  $f(x)$ .

a)  $f(x) = 0$  at  $x = 2$

$$\lim_{x \rightarrow 4} f(x) = DNE$$

*crosses y-axis at  $y = -1$*

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = -2$$

$$\lim_{x \rightarrow 4^+} f(x) = -\infty$$

$f(x)$  DNE at  $x = -1$  and  $x = 4$

b)  $\lim_{x \rightarrow 2} f(x) = -1$

$$\lim_{x \rightarrow 4^+} f(x) = -\infty$$

$$\lim_{x \rightarrow 4^-} f(x) = \infty$$

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

$$\lim_{x \rightarrow -\infty} f(x) = 2$$

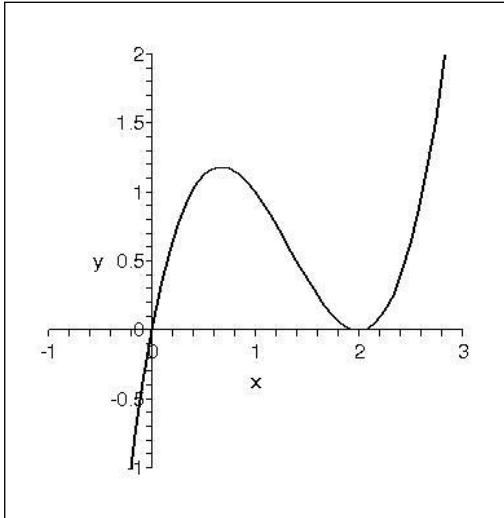


## Section 2.4: Rates of Change and Tangent Lines

- 1) Find the average rate of change for the function  $f(x) = 5x^2 - 3$  over the interval  $[0, 2]$ .
  
- 2) Find the average rate of change for the function  $f(x) = e^{2x-3} - x$  over the interval  $[1, 2]$ . Leave answer in terms of  $e$ .
  
- 3) Find the average rate of change for the function  $f(x) = \cos x - 4$  over the interval  $[0, \pi]$ .
  
- 4) Consider the function  $f(x) = x^2 - 4x$ .
  - a) Find the slope of the line tangent to  $f(x)$  at the point  $(1, -3)$ .
  
  - b) Write the equation for the line tangent to  $f(x)$  at  $(1, -3)$ .
  
  - c) Write the equation for the line normal  $f(x)$  at  $(1, -3)$ .
  
- 5) Consider the function  $f(x) = \sqrt{x+4}$ .
  - a) Write the equation for the line tangent to  $f(x)$  at  $x = 0$ .
  
  - b) Write the equation for the line normal to  $f(x)$  at  $x = 0$ .
  
- 6) A coffee shop opens at 5:00 am. The number of coffee cups,  $f(x)$ , sold per hour can be modeled by the function  $f(x) = 3x^3 - 2x^2 + 6$ , where  $x$  is the number of hours the shop has been open. Find the average rate of change between the hours of 7:00am and 11:00am and **explain** what this rate represents.

7) Explain the difference between average rate of change and instantaneous rate of change.

8) Below is a graph of the function  $f(x) = x(x-2)^2$ .



a) Draw a line tangent to the  $f(x)$  at  $x = 1$ . Is the slope of the tangent line positive or negative?

b) Draw a secant line through the points  $(1, 1)$  and  $(2.8, 1.792)$  on  $f(x)$ . Draw another secant line through the points  $(1, 1)$  and  $(0.6, 1.176)$ . Which secant line is the best approximation of the line tangent to  $f(x)$  at  $(1, 1)$ ? Why?

9) Determine the slope of each curve at  $x = a$ .

a)  $y = x^2 - x - 2$

b)  $y = \frac{1}{x+2}$

10) At what points, if any, are the tangents to the graph  $f(x) = x^2 - 3x$  horizontal?

11) Clara is hovering above the asteroid Elmy. She drops a rock, and the position (in feet) of the rock is modeled by the equation  $f(x) = -3x^2 + 800$  ( $x$  measured in seconds)

a) What is the equation for the instantaneous velocity of the rock?

b) What is the instantaneous velocity of the rock at 8 seconds?

### Section 3.1: Derivative of a Function

1) What are the two definitions you can use to find a derivative?

2) Use the definition of a derivative to find  $f'(x)$  for each function:

a)  $f(x) = \frac{2}{x-3}$  at  $x = 2$ .

b)  $f(x) = 4x^2 + x - 5$

c)  $f(x) = \sqrt{x-3}$

3) Find the equations of the tangent and normal lines at  $x = 2$ , given the information below:

$$f(2) = 6, f'(0) = 4 \text{ and } f'(2) = -3$$

4) Find the left-hand and right-hand derivatives in order to determine if the derivative exists when  $x = 1$  for the

$$\text{function } f(x) = \begin{cases} 2x^2 + 1, & x < 1 \\ 3x + 6, & x \geq 1 \end{cases}$$

5) Find the derivative of  $y = 2x^2 - 13x + 5$  and use it to find the equation of the line tangent to the curve at  $x = 3$ .